

Roll No.

BCA-405(N)

B. C. A. (Fourth Semester)
EXAMINATION, May, 2017

(New Course)

Paper Fifth

MATHEMATICS—III

Time : Three Hours]

[Maximum Marks : 75

Note : Section A is compulsory. Attempt any *ten* questions from Section B and attempt any *one* question from Section C.

Section—A

15

1. (a) Find Fourier series of $f(x) = x$, $0 < x < 2\pi$.
(b) Simplify $(5 - i)^2 + (2i + 3) + 12(i - 1)$.
2. (a) Solve the ordinary differential equation $y'' - 4y = e^x + \sin 3x$.
(b) Test the convergence of series $\sum_{n=1}^{\infty} (\sqrt{n^2 + 1} - n)$.

Section—B

45

3. Solve :

$$\frac{dy}{dx} = \frac{y + x + 1}{x + y - 1}$$

4. Find all values of $(i)^{\frac{1}{3}}$.

5. If $\vec{v} = x^2 y^2 \hat{i} + 2xy \hat{j} + (y^2 - xy) \hat{k}$, find :

(i) $\text{div } \vec{v}$

(ii) $\text{curl } \vec{v}$

6. Find the limit of sequence $\{S_n\}_{n=1}^{\infty}$ where

$$S_n = \frac{3n}{n + 5\sqrt{n}}.$$

7. Find directional derivative of function $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at $P(1, 1, 2)$ in the direction of vector $\vec{a} = \hat{i} - 2\hat{k}$.

8. Find Fourier series representation of function $f(x) = \begin{cases} 2 & -2 < x < 0 \\ x & 0 < x < 2 \end{cases}$.

9. Test for the convergence of series :

$$\frac{1}{3+1} + \frac{2}{3^2+1} + \frac{2^2}{3^3+1} + \dots$$

10. Prove that :

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$$

11. Solve :

$$(1+x^2)y' + 2yx = \cos x$$

12. If $z_1 = 2 + 3i$ and $z_2 = 1 + i$ find $\frac{2z_1}{z_2}$.

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13. Find the solution of differential equation
 $(x^2 + y^2 + 1)dx - 2xy dy = 0$.

14. Let $\vec{r} = \vec{a} \sin \omega t + \vec{b} \cos \omega t$, where $\vec{a}, \vec{b}$ and ω are constants. Prove that :

$$\vec{r} \times \frac{d\vec{r}}{dt} = -\omega (\vec{a} \times \vec{b})$$

15. Find the solution of $(x^2 D^2 - 2xD - 4)y = x^2$ where

$$D \equiv \frac{d}{dx}$$

Section—C

15

16. For the function $f(x) = e^x$, find half range :

- (i) sine series in $0 < x < \pi$
- (ii) cosine series in $0 < x < \pi$.

17. Find value of 'a' such that vector field

$$\vec{F} = 2xyz^3 \hat{i} + ax^2y^3 \hat{j} + 3x^2yz^2 \hat{k}$$

is irrotational. Hence find scalar function of such that $\vec{F} = \nabla \phi$.

18. (i) Discuss the convergence of series :

$$1 + \frac{2x}{2!} + \frac{3^2 x^2}{3!} + \frac{4^3 x^3}{4!} + \dots, x > 0$$

- (ii) Prove that the sequence $\left\{ \frac{2n-7}{3n+2} \right\}_0^\infty$ is monotonic

increasing and bounded. Also, find the limit of sequence.

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2,800